Abstract:
Markov processes are often employed to model credit rating transitions if continuous-time rating migration data are available. However, such data are often very expensive whereas discrete-time data are available for free. Thus, an important branch in the credit risk literature concentrates on the estimation of discretely-observed Markov processes. For this problem, an analytical expression of the observed Fisher information matrix based on a general relationship between the Fisher information matrix and the expectation-maximization algorithm can be derived. Thereby, an expression that partial derivatives of a matrix exponential function can again be represented by single matrix exponential functions is used. However, by having this result available, the indirection of deriving the Fisher information matrix by components of the expectation-maximization algorithm is no longer necessary. Instead, the information matrix can be directly obtained as the negative second order derivative of the discretely observed Markov process log likelihood function. This result is of distinctly lower complexity than the already existing expression so that the computing time for confidence intervals can be reduced to less than one half. Moreover, an analytical expression for the Delta method of matrix exponential transformations of Markov generator matrices is derived, where an intuitive interpretation of the parameters on the level of (rating) transition frequencies is given.